SAMPLE QUESTION PAPER (2024 - 25)

CLASS- XII

SUBJECT: Mathematics (041)

Time: 3 Hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are **compulsory.**

(ii) This Question paper is divided into **five** Sections - **A, B, C, D** and **E.**

(iii) In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.

(iv) In **Section B,** Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.

(v) In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.

(vi) In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.

(vii) In **Section E,** Questions no. **36** to **38** are **Case study-based questions,** carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION-A $[1\times 20=20]$

(***This section comprises of multiple choice questions (MCQs) of 1 mark each***)

Select the correct option (Question 1 - Question 18):

Q.1. If for a square matrix A, $A.(adjA) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix}$, then the value of $|A|+|adj\,A|$ is equal to:

(A) 1 (B) $2025+1$ (C) $(2025)^2+45$ (D) $2025+(2025)^2$

Q.2. Assume X, Y, Z, W and P are matrices of order $2\times n$, $3\times k$, $2\times p$, $n\times 3$ and $p\times k$, respectively. Then the restriction on n, k and p so that $PY + WY$ will be defined are:

(A) $k = 3, p = n$ (B) k **is arbitrary,** $p = 2$

(C) p **is arbitrary,** $k = 3$ (D) $k = 2, p = 3$

Q.3. The interval in which the function f defined by $f(x) = e^x$ is strictly increasing, is

(A) $[1, \infty)$ (B) $(-\infty, 0)$ (C) $(-\infty, \infty)$ (D) $(0, \infty)$

Q.4. If $\boldsymbol{A}$ **and** $\boldsymbol{B}$ are non-singular matrices of same order with $\boldsymbol{det(A) = 5}$, then $\boldsymbol{det(B^{-1}AB)^2}$ is equal to

(A) 5 (B) 5^2 (C) 5^4 (D) 5^5

Q.5. The value of $'\boldsymbol{n}'$, such that the differential equation $\boldsymbol{x^n \frac{dy}{dx} = y(logy - logx + 1)}$;

$(\textbf{where } \boldsymbol{x, y} \in \boldsymbol{R^+})$ is homogeneous, is

(A) **0** (B) **1** (C) **2** (D) **3**

Q.6. If the points $(\boldsymbol{x_1, y_1})$, $(\boldsymbol{x_2, y_2})$ **and** $(\boldsymbol{x_1 + x_2, y_1 + y_2})$ are collinear, then $\boldsymbol{x_1 y_2}$ is equal to

(A) $x_2 y_1$ (B) $x_1 y_1$ (C) $x_2 y_2$ (D) $x_1 x_2$

Q.7. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew-symmetric matrix then the value of $\boldsymbol{a+b+c} =$

(A)1 (B) **2** (C) **3** (D) **4**

Q.8. For any two events $\boldsymbol{A}$ and $\boldsymbol{B}$, if $\boldsymbol{P(\overline{A}) = \frac{1}{2}, P(\overline{B}) = \frac{2}{3}}$ and $\boldsymbol{P(A \cap B) = \frac{1}{4}}$, then $P\left(\bar{A}/\bar{B}\right)$ equals:

(A) $\frac{3}{8}$ (B) $\frac{8}{9}$ (C) $\frac{5}{8}$ (D) $\frac{1}{4}$

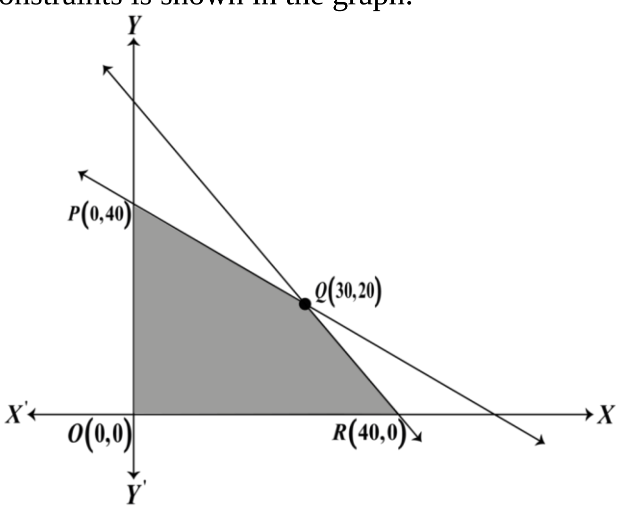
Q.9. The value of α if the angle between $\vec{p} = 2\alpha^2\hat{\imath} - 3\alpha\hat{\jmath} + \hat{k}$ and $\vec{q} = \hat{\imath} + \hat{\jmath} + \alpha\hat{k}$ is obtuse, is

(A) $R - [0, 1]$ (B) (0, 1) (C) [0, ∞) (D) [1, ∞)

Q.10. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}| =$

(A) **3** (B) **4** (C) **5** (D) **8**

Q.11. For the linear programming problem (**LPP**), the objective function is $\boldsymbol{Z = 4x + 3y}$ and the feasible region determined by a set of constraints is shown in the graph:



(Note*: The figure is not to scale.)*

Which of the following statements is true?

(A) Maximum value of Z is at $R(40,0)$.

(B) Maximum value of Z is at $Q(30,20)$.

(C) Value of Z at $R(40,0)$ is less than the value at $P(0,40)$.

(D) The value of Z at $Q(30,20)$ is less than the value at $R(40,0)$.

Q.12. $\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}$ equals

(A) $-\frac{1}{2x^2}\sqrt{1+x^4}+c$

(B) $\frac{1}{2x}\sqrt{1+x^4}+c$

(C) $-\frac{1}{4x}\sqrt{1+x^4}+c$

(D) $\frac{1}{4x^2}\sqrt{1+x^4}+c$

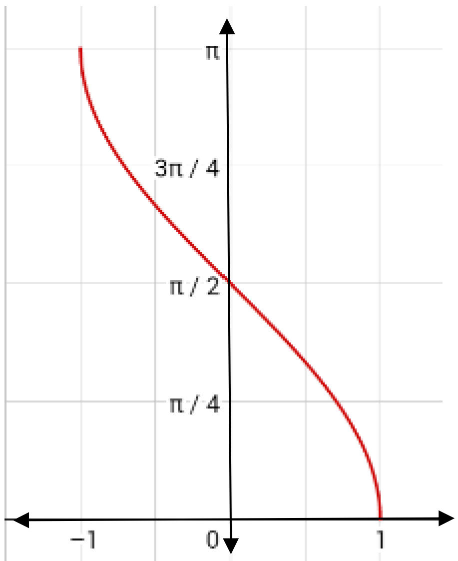
Q.13. $\int_0^{2\pi} cosec^7 x\, dx =$

(A) **0** (B) **1** (C) **4** (D) **2π**

Q.14. What is the general solution of the differential equation $e^{y'} = x$?

(A) $y = xlogx + c$ (B) $y = xlogx - x + c$ (C) $y = xlogx + x + c$ (D) $y = x + c$

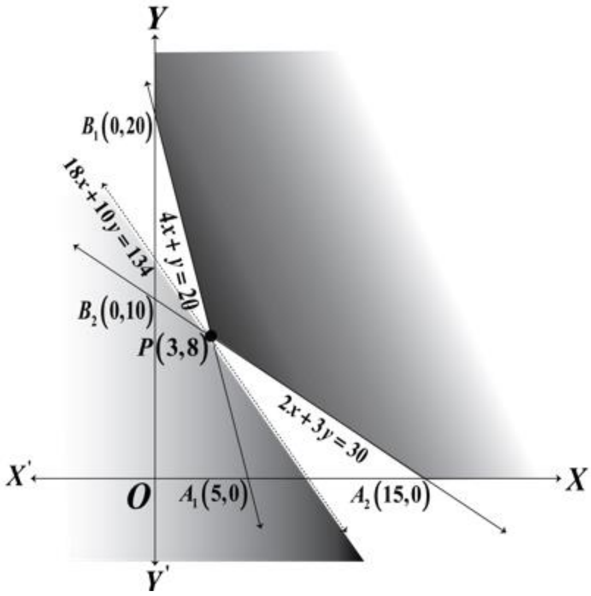
Q.15. The graph drawn below depicts



(A) y = $sin^{-1} x$ (B) y = $cos^{-1} x$ (C) y = $cos\,e\,c^{-1} x$ (D) y = $cot^{-1} x$

Q.16. A linear programming problem (**LPP**) along with the graph of its constraints is shown below.

The corresponding objective function is: $Z = 18x + 10y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3,8)$,



(Note*: The figure is not to scale.)*

The optimal solution of the above linear programming problem __________.

(A) does not exist as the feasible region is unbounded.

(B) does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

(C) exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region.

(D) exists as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

Q.17. The function $f: R \to Z$ defined by $f(x) = [x]$; where $[.]$ denotes the greatest integer function, is

(A) Continuous at $x = 2.5$ but not differentiable at $x = 2.5$

(B) Not Continuous at $x = 2.5$ but differentiable at $x = 2.5$

(C) Not Continuous at $x = 2.5$ and not differentiable at $x = 2.5$

(D) Continuous as well as differentiable at $x = 2.5$

Q.18. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = 4y$. Then the area (in sq units) of the region bounded by parabola $x^2 = 4y$ and the line $y = 4$ is

(A) $\frac{32}{3}$ (B) $\frac{64}{3}$ (C) $\frac{128}{3}$ (D) $\frac{256}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Q.19. Assertion (A): Consider the function defined as $f(x) = |x| + |x-1|,\ x \in R$. Then $f(x)$ is not differentiable **at $x = 0$ and $x = 1$**.

Reason (R): Suppose f be defined and continuous on (a,b) and $c \in (a,b)$, then $f(x)$ is not differentiable at $x = c$ if $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} \neq \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$.

Q.20. Assertion (A): The function $f: R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\} \to (-\infty,-1] \cup [1,\infty)$ defined by $f(x) = \sec x$ is not one-one function in its domain.

Reason (R): The line $y = 2$ meets the graph of the function at more than one point.

SECTION B $[2 \times 5 = 10]$

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

Q.21. If $cot^{-1}(3x+5) > \frac{\pi}{4}$, then find the range of the values of x.

Q.22. The cost (in rupees) of producing x items in factory, each day is given by

$$C(x) = 0.00013x^3 + 0.002\,x^2 + 5x + 2200$$

Find the marginal cost when 150 items are produced.

Q.23. (a) Find the derivative of $\tan^{-1} x$ with respect to $\log x$; (where $x \in (1,\infty)$).

OR

Q.23. (b) Differentiate the following function with respect to x : $(cos\, x)^x$; $\left(\text{where } x \in \left(0,\frac{\pi}{2}\right)\right)$.

Q.24. (a) If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.

OR

Q.24. (b) A person standing at $O(0,0,0)$ is watching an aeroplane which is at the coordinate point $A(4,0,3)$. At the same time he saw a bird at the coordinate point $B(0,0,1)$. Find the angles which $\overrightarrow{BA}$ makes with the x,y and z axes.

Q.25. The two co-initial adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram.

SECTION C $\left[3\times 6=18\right]$

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

Q.26. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

Q.27. According to a psychologist, the ability of a person to understand spatial concepts is given by

$A=\frac{1}{3}\sqrt{t}$, where t is the age in years, $t\in\left[5,18\right]$. Show that the rate of increase of the ability to understand spatial concepts decreases with age in between 5 and 18.

Q.28. (a) An ant is moving along the vector $\vec{l_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$. Few sugar crystals are kept along the vector $\vec{l_2} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ which is inclined at an angle θ with the vector $\vec{l_1}$. Then find the angle θ. Also find the scalar projection of $\vec{l_1}$ on $\vec{l_2}$.

OR

Q.28. (b) Find the vector and the cartesian equation of the line that passes through $(-1, 2, 7)$ and is perpendicular to the lines $\vec{r} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 7\hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$.

Q.29. (a) Evaluate: $\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$; (where $x>1$).

OR

Q.29. (b) Evaluate : $\int_0^1 x(1-x)^n dx$; (where $n \in N$).

Q.30. Consider the following Linear Programming Problem:

Minimise $Z = x+2y$

Subject to $2x+y\geq 3,\ x+2y\geq 6,\ x,y\geq 0.$

Show graphically that the minimum of Z occurs at more than two points

Q.31. (a) The probability that it rains today is **0.4**. If it rains today, the probability that it will rain tomorrow is **0.8**. If it does not rain today, the probability that it will rain tomorrow is **0.7**. If

P_1: denotes the probability that it does not rain today.

P_2: denotes the probability that it will not rain tomorrow, if it rains today.

P_3: denotes the probability that it will rain tomorrow, if it does not rain today.

P_4: denotes the probability that it will not rain tomorrow, if it does not rain today.

(i) Find the value of $P_1\times P_4 - P_2\times P_3$. $\left[2\,Marks\right]$

(ii) Calculate the probability of raining tomorrow. $\left[1\,Mark\right]$

OR

Q.31. (b) A random variable X can take all non – negative integral values and the probability that X takes the value $\boldsymbol{r}$ is proportional to 5^{-r}. Find $\boldsymbol{P(X<3)}$.

SECTION D

$[5\times4=20]$

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q.32. Draw the rough sketch of the curve $y = 20\cos 2x$; $\left(\text{where} \frac{\pi}{6} \le x \le \frac{\pi}{3}\right)$.

U**sing integration,** find the area of the region bounded by the curve y = 20 cos2x from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$ and the $\boldsymbol{x}$ – axis.

Q.33. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; $(where\ 0 \le x \le 14\ and\ a, b, c \in R\ and\ a \ne 0)$ with respect to a XY-coordinate system in the vertical plane. The ball passes through the points $\boldsymbol{(2,15)}$ **,** $\boldsymbol{(4,25)}$ **and** $\boldsymbol{(14,15)}$. Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.

Q.34. (a) If $f: R \to R$ is defined by $f(x) = |x|^3$, show that $f''(x)$ exists for all real $\boldsymbol{x}$ and find it.

OR

Q.34. (b) If $(x-a)^2 + (y-b)^2 = c^2$, for some $\boldsymbol{c>0}$**,** prove that $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Q.35. (a) Find the shortest distance between the lines $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ whose vector equations are

$$\vec{r} = \left(-\hat{\imath} - \hat{\jmath} - \hat{k}\right) + \lambda\left(7\hat{\imath} - 6\hat{\jmath} + \hat{k}\right) \text{ and } \vec{r} = \left(3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}\right) + \mu\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$$

where λ and μ are parameters.

OR

Q.35. (b) Find the image of the point $\boldsymbol{(1,2,1)}$ with respect to the line $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3}$. Also find the equation of the line joining the given point and its image.

<u>SECTION- E</u>

$[4\times3=12]$

(This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)

Case Study-1

Q.36. Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension **25** $\boldsymbol{cm}$ **by 40** $\boldsymbol{cm}$ to make container packets without top. Let $\boldsymbol{x\,cm}$ be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.

Based on the above information answer the following questions.

(i) Express the volume (V) of each container as function of x only. $[1\,Mark]$

(ii) Find $\frac{dV}{dx}$ $[1\,Mark]$

(iii) (a) For what value of x, the volume of each container is maximum? $[2\,Marks]$

OR

(iii) (b) Check whether V has a point of inflection at x = $\frac{65}{6}$ or not? $[2\,Marks]$

Case Study-2

Q.37. An organization conducted bike race under 2 different categories-boys and girls. In all, there were **250** participants. Among all of them finally three from Category **1** and two from Category **2** were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}, G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible? $[1\,Mark]$

(ii) Write the smallest equivalence relation on **G**. $[1\,Mark]$

(iii) (a) Ravi defines a relation from **B to B** as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive. $[2\,Marks]$

OR

(iii) (b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; $(\text{where} 0 \le x \le 20\sqrt{2} \& 0 \le y \le 200)$, then state whether the track represents a one-one and onto function or not. (Justify). $[2\,Marks]$

Case Study- 3

Q.38. Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage –II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I(simultaneously).

Assume that all the birds have equal chances of flying.

On the basis of the above information, answer the following questions:-

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I. $\left[\textbf{2} \, \textbf{\textit{Marks}}\right]$

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II? $\left[\textbf{2} \, \textbf{\textit{Marks}}\right]$

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MARKING SCHEME

CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Q no.	ANS	HINTS/SOLUTION
1.	**(D)**	For a square matrix $\boldsymbol{A}$ of order $\boldsymbol{n \times n}$, we have $\boldsymbol{A.(adj\,A) = \lvert A \rvert I_n}$, where $\boldsymbol{I_n}$ is the identity matrix of order $\boldsymbol{n \times n}$. So, $\boldsymbol{A.(adj\,A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix} = 2025 I_3 \Rightarrow \lvert A \rvert = 2025}$ **&** $\boldsymbol{\lvert adj\,A \rvert = \lvert A \rvert^{3-1} = (2025)^2}$ $\boldsymbol{\therefore \lvert A \rvert + \lvert adj\,A \rvert = 2025 + (2025)^2}$.
2.	**(A)**	P ↓ Order $p \times k$; Y ↓ Order $3 \times k$ For PY to exist $k = 3$ Order of $PY = p \times k$ W ↓ Order $n \times 3$; Y ↓ Order $3 \times k$ Order of $WY = n \times k$ For $PY + WY$ to exist order(PY) = order(WY) $\therefore p = n$
3.	**(C)**	$y = e^x \Rightarrow \frac{dy}{dx} = e^x$ In the domain (R) of the function, $\frac{dy}{dx} > 0$, hence the function is strictly increasing in $(-\infty, \infty)$
4.	**(B)**	$\boldsymbol{\lvert A \rvert = 5, \lvert B^{-1}AB \rvert^2 = (\lvert B^{-1} \rvert \lvert A \rvert \lvert B \rvert)^2 = \lvert A \rvert^2 = 5^2}$.
5.	**(B)**	A differential equation of the form $\boldsymbol{\frac{dy}{dx} = f(x, y)}$ is said to be homogeneous, if $\boldsymbol{f(x, y)}$ is a homogeneous function of degree 0. Now, $\boldsymbol{x^n \frac{dy}{dx} = y\left(\log_e \frac{y}{x} + \log_e e\right) \Rightarrow \frac{dy}{dx} = \frac{y}{x^n}\left(\log_e e.\left(\frac{y}{x}\right)\right) = f(x, y); (Let)}$. $\boldsymbol{f(x, y)}$ will be a homogeneous function of degree **0,** if $\boldsymbol{n = 1}$.
6.	**(A)**	**Method 1:** (***Short cut***) When the points $\boldsymbol{(x_1, y_1), (x_2, y_2)}$ and $\boldsymbol{(x_1 + x_2, y_1 + y_2)}$ are collinear in the Cartesian plane then $\boldsymbol{\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - (x_1 + x_2) & y_1 - (y_1 + y_2) \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ -x_2 & -y_2 \end{vmatrix} = (-x_1y_2 + x_2y_2 + x_2y_1 - x_2y_2) = 0}$ $\boldsymbol{\Rightarrow x_2y_1 = x_1y_2}$.

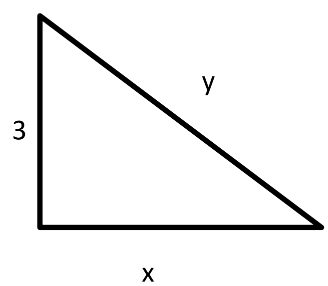
<table>
<tr><td></td><td></td><td>Method 2:
When the points $(x_1, y_1),(x_2, y_2)$ and (x_1+x_2, y_1+y_2) are collinear in the Cartesian plane then
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1+x_2 & y_1+y_2 & 1 \end{vmatrix} = 0$$
$\Rightarrow 1.(x_2y_1 + x_2y_2 - x_1y_2 - x_2y_2) - 1(x_1y_1 + x_1y_2 - x_1y_1 - x_2y_1) + (x_1y_2 - x_2y_1) = 0$
$\Rightarrow x_2y_1 = x_1y_2$.</td></tr>
<tr><td>7.</td><td>(A)</td><td>$$A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$$
When the matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$;
$\Rightarrow c = -2; a = 0$ and $b = 3$
So , $a + b + c = 0 + 3 - 2 = 1$.</td></tr>
<tr><td>8.</td><td>(C)</td><td>$P(\overline{A}) = \frac{1}{2}; P(\overline{B}) = \frac{2}{3}; P(A \cap B) = \frac{1}{4}$
$\Rightarrow P(A) = \frac{1}{2}; P(B) = \frac{1}{3}$
We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$
$$P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})} = \frac{1 - \frac{7}{12}}{\frac{2}{3}} = \frac{5}{8}.$$</td></tr>
<tr><td>9.</td><td>(B)</td><td>For obtuse angle, $\cos\theta < 0$ => $\vec{p}.\vec{q} < 0$
$2\alpha^2 - 3\alpha + \alpha < 0$ => $2\alpha^2 - 2\alpha < 0$ => $\alpha \in (0, 1)$</td></tr>
<tr><td>10.</td><td>(C)</td><td>$|\vec{a}| = 3, |\vec{b}| = 4, |\vec{a} + \vec{b}| = 5$
We have , $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right) = 2(9+16) = 50 \Rightarrow |\vec{a} - \vec{b}| = 5$.</td></tr>
<tr><td>11.</td><td>(B)</td><td><table><tr><td>Corner point</td><td>Value of the objective function $Z = 4x + 3y$</td></tr><tr><td>1. $O(0,0)$</td><td>$z = 0$</td></tr><tr><td>2. $R(40,0)$</td><td>$z = 160$</td></tr><tr><td>3. $Q(30,20)$</td><td>$z = 120 + 60 = 180$</td></tr><tr><td>4. $P(0,40)$</td><td>$z = 120$</td></tr></table>
Since , the feasible region is bounded so the maximum value of the objective function $z = 180$ is at $Q(30,20)$.</td></tr>
</table>

12.	**(A)**	$\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{1}{2}}}$ (Let $1+x^{-4} = 1+\frac{1}{x^4} = t,\ dt = -4x^{-5}dx = -\frac{4}{x^5}dx \Rightarrow \frac{dx}{x^5} = -\frac{1}{4}dt$) $= -\frac{1}{4}\int \frac{dt}{t^{\frac{1}{2}}} = -\frac{1}{4} \times 2 \times \sqrt{t} + c$, where $'c'$ denotes any arbitrary constant of integration. $= -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + c = -\frac{1}{2x^2}\sqrt{1+x^4} + c$
13.	**(A)**	We know, $\int_0^{2a} f(x)dx = 0$, **if** $f(2a-x) = -f(x)$ Let $f(x) = \operatorname{cosec}^7 x$. Now, $f(2\pi - x) = \operatorname{cosec}^7(2\pi - x) = -\operatorname{cosec}^7 x = -f(x)$ $\therefore \int_0^{2\pi} \operatorname{cosec}^7 x\, dx = 0;$ Using the property $\int_0^{2a} f(x)dx = 0$, **if** $f(2a-x) = -f(x)$.
14.	**(B)**	The given differential equation $e^{y'} = x \Rightarrow \frac{dy}{dx} = \log x$ $dy = \log x\, dx \Rightarrow \int dy = \int \log x\, dx$ $y = x\log x - x + c$ hence the correct option is **(B)**.
15.	**(B)**	The graph represents $y = \cos^{-1} x$ whose domain is $[-1,1]$ and range is $[0,\pi]$.
16.	**(D)**	Since the inequality $Z = 18x + 10y < 134$ has no point in common with the feasible region hence the minimum value of the **objective function** $Z = 18x + 10y$ is 134 at $P(3,8)$.
17.	**(D)**	The graph of the function $f: R \to R$ defined by $f(x) = [x]$; (**where** $[.]$ **denotes** $G.I.F$) is a straight line $\forall x \in (2.5 - h, 2.5 + h)$, $'h'$ is an infinitesimally small positive quantity. Hence, the function is continuous and differentiable at $x = 2.5$.
18.	**(B)**	**The required region is symmetric about the** $y-$ axis. So, required area (in sq units) is $= \left\| 2\int_0^4 2\sqrt{y}dy \right\| = 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{64}{3}$.
19.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**
20.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**

<u>Section –B</u>

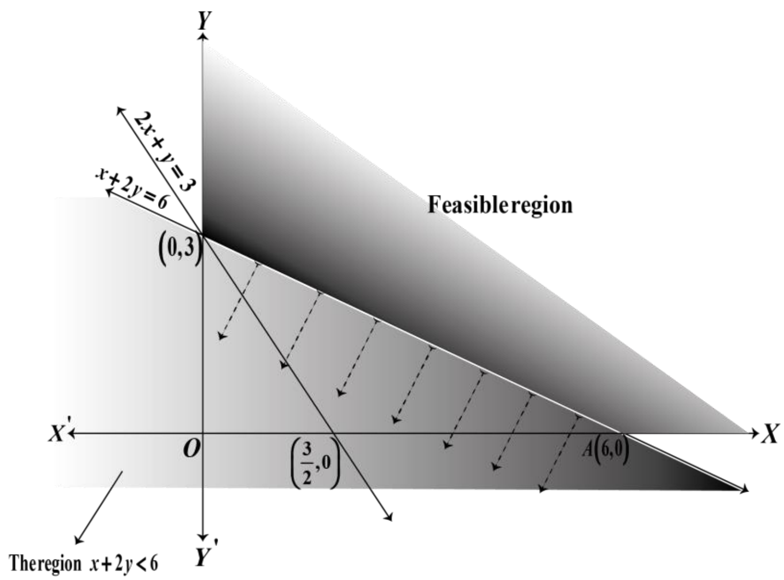
[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$\cot^{-1}(3x+5) > \frac{\pi}{4} = \cot^{-1} 1$ =>3x + 5 < 1 (as $\cot^{-1}x$ is strictly decreasing function in its domain) => 3x < – 4 => $x < -\frac{4}{3}$ $\therefore x \in \left(-\infty, -\frac{4}{3}\right)$	$\frac{1}{2}$ $\frac{1}{2}$ 1
22.	The marginal cost function is $C'(x) = 0.00039x^2 + 0.004x + 5$. $C'(150) =$ ₹ 14.375 .	1 1
23.(a)	$y = \tan^{-1} x$ and $z = \log_e x$. Then $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = \frac{1}{x}$ So, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ $= \frac{\frac{1}{1+x^2}}{\frac{1}{x}} = \frac{x}{1+x^2}$.	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR 23.(b)	Let $y = (\cos x)^x$. Then, $y = e^{x \log_e \cos x}$ On differentiating both sides with respect to x, we get $\frac{dy}{dx} = e^{x\log_e \cos x} \frac{d}{dx}(x \log_e \cos x)$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{\log_e \cos x \frac{d}{dx}(x) + x\frac{d}{dx}(\log_e \cos x)\right\}$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{\log_e \cos x + x.\frac{1}{\cos x}(-\sin x)\right\} \Rightarrow \frac{dy}{dx} = (\cos x)^x(\log_e \cos x - x\tan x)$.	 $\frac{1}{2}$ $\frac{1}{2}$ 1
24.(a)	We have $\vec{b} + \lambda\vec{c} = (-1+3\lambda)\hat{\imath} + (2+\lambda)\hat{\jmath} + \hat{k}$ $(\vec{b} + \lambda\vec{c}).\vec{a} = 0 \Rightarrow 2(-1+3\lambda) + 2(2+\lambda) + 3 = 0$ $\lambda = -\frac{5}{8}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
OR 24.(b)	$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (4\hat{\imath} + 3\hat{k}) - \hat{k} = 4\hat{\imath} + 2\hat{k}$	$\frac{1}{2}$

	$\widehat{BA} = \frac{4}{2\sqrt{5}}\hat{\imath} + \frac{2}{2\sqrt{5}}\hat{k} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{k}$ So, the angles made by the vector $\overrightarrow{BA}$ with the $\boldsymbol{x}, \boldsymbol{y}$ and the $\boldsymbol{z}$ axes are respectively $cos^{-1}\left(\frac{2}{\sqrt{5}}\right), \frac{\pi}{2}, cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$.	**$\frac{1}{2}$** **1**
25.	$\overrightarrow{d_1} = \vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a} - \vec{b} = -6\hat{\jmath} - 8\hat{k}$ Area of the parallelogram = $\frac{1}{2}\left\lvert\overrightarrow{d_1} \times \overrightarrow{d_2}\right\rvert = \frac{1}{2}\left\lVert\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{matrix}\right\rVert = 2\left\lvert\hat{\imath} + 8\hat{\jmath} - 6\hat{k}\right\rvert$ Area of the parallelogram = $2\sqrt{101}$ sq. units.	**$\frac{1}{2}$** **1** **$\frac{1}{2}$**
	Section –C **[This section comprises of solution short answer type questions (SA) of 3 marks each]**	
26.	 $x^2 + 3^2 = y^2$ $When\ y = 5\ then\ x = 4, now\ 2x\ \frac{dx}{dt} = 2y\ \frac{dy}{dt}$ $4\ (200) = 5\ \frac{dy}{dt}$ => $\frac{dy}{dt} = 160$ cm/s	**$\frac{1}{2}$** **$\frac{1}{2}$** **1** **1**
27.	$A = \frac{1}{3}\sqrt{t} \therefore \frac{dA}{dt} = \frac{1}{6}t^{-\frac{1}{2}} = \frac{1}{6\sqrt{t}}; \forall t \in (5,18)$ $\frac{dA}{dt} = \frac{1}{6\sqrt{t}} \therefore \frac{d^2A}{dt^2} = -\frac{1}{12t\sqrt{t}}$ So, $\frac{d^2A}{dt^2} < 0, \forall t \in (5,18)$ This means that the rate of change of the ability to understand spatial concepts decreases (slows down) with age.	**1** **1** **$\frac{1}{2}$** **$\frac{1}{2}$**
28(a)	**(i)** $\boldsymbol{\theta = \cos^{-1}\left(\frac{\overrightarrow{l_1}.\overrightarrow{l_2}}{\lvert\overrightarrow{l_1}\rvert.\lvert\overrightarrow{l_2}\rvert}\right) = \cos^{-1}}\left(\frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\lvert(\hat{\imath}-2\hat{\jmath}+3\hat{k})\rvert\lvert(3\hat{\imath}-2\hat{\jmath}+\hat{k})\rvert}\right)$ $\boldsymbol{= cos^{-1}\left(\frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}\right) = cos^{-1}\left(\frac{10}{14}\right) = cos^{-1}\left(\frac{5}{7}\right)}$. **(ii)** Scalar projection of $\overrightarrow{l_1}$ on $\overrightarrow{l_2}$ $= \frac{\overrightarrow{l_1}.\overrightarrow{l_2}}{\lvert\overrightarrow{l_2}\rvert} = \frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\lvert(3\hat{\imath}-2\hat{\jmath}+\hat{k})\rvert}$ $= \frac{3+4+3}{\sqrt{9+4+1}} = \frac{10}{\sqrt{14}}$.	1 **$\frac{1}{2}$** 1 **$\frac{1}{2}$**

28(b)	Line perpendicular to the lines $\vec{r} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 7\hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$. has a vector parallel it is given by $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$ $\therefore$ equation of line in vector form is $\vec{r} = -\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + a(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k})$ And equation of line in cartesian form is $\frac{x+1}{10} = \frac{y-2}{5} = \frac{z-7}{-4}$	1 1 1
29.(a)	$\int \left\{\frac{1}{log_e x} - \frac{1}{(log_e x)^2}\right\} dx$ $= \int \frac{dx}{log_e x} - \int \frac{1}{(log_e x)^2} dx = \frac{1}{log_e x}\int dx - \int \left\{\frac{d}{dx}\left(\frac{1}{log_e x}\right)\int dx\right\} dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2}\frac{1}{x}.x.dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2} dx - \int \frac{dx}{(log_e x)^2} = \frac{x}{log_e x} + c;$ where $'c'$ is any arbitary constant of integration.	**1** **1** **1**
OR **29.(b)**	$\int_0^1 x(1-x)^n dx$ $= \int_0^1 (1-x)\{1-(1-x)\}^n dx, \left(as, \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$ $= \int_0^1 x^n (1-x)dx$ $= \int_0^1 x^n dx - \int_0^1 x^{n+1} dx$ $= \frac{1}{n+1}[x^{n+1}]_0^1 - \frac{1}{n+2}[x^{n+2}]_0^1$ $= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}.$	**1** $\frac{1}{2}$ $\frac{1}{2}$ **1**
30.	The feasible region determined by the constraints, $2x + y \geq 3,\ x + 2y \geq 6,\ x \geq 0, y \geq 0$ is as shown.	

1



The corner points of the **unbounded** feasible region are $A(6,0)$ and $B(0,3)$.

The values of Z at these corner points are as follows:

Corner point	Value of the objective function $Z = x + 2y$
$A(6,0)$	**6**
$B(0,3)$	**6**

1

We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z = 6$. $\frac{1}{2}$

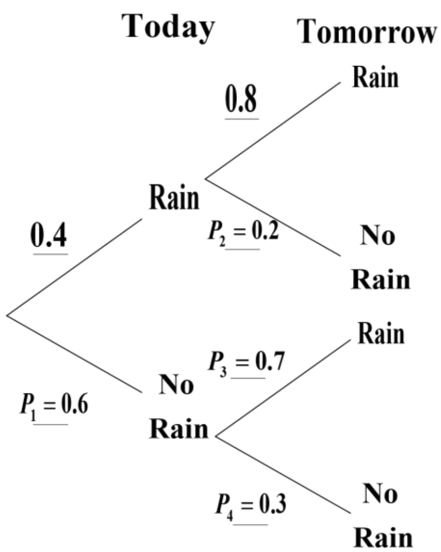
It can be seen that the value of Z at points **A** and **B** is same. If we take any other point on the line $x + 2y = 6$ such as (2,2) on line $x + 2y = 6$, then $Z = 6$.

Thus, the minimum value of Z occurs for more than **2** points, and is equal to 6. $\frac{1}{2}$

31.(a) Since the event of raining today and not raining today are complementary events so if the probability that it rains today is 0.4 then the probability that it does not rain today is $1 - 0.4 = 0.6 \Rightarrow P_1 = 0.6$

If it rains today, the probability that it will rain tomorrow is 0.8 then the probability that it will not rain tomorrow is $\mathbf{1-0.8=0.2}$.

If it does not rain today, the probability that it will rain tomorrow is 0.7 then the probability that it will not rain tomorrow is $\mathbf{1-0.7=0.3}$



(i) $\boldsymbol{P_1 \times P_4 - P_2 \times P_3 = 0.6\times 0.3 - 0.2\times 0.7 = 0.04.}$ **1**

(ii) Let $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ be the events that it will rain today and it will not rain today respectively. **1**

$\boldsymbol{P(E_1)=0.4}$ **&** $\boldsymbol{P(E_2)=0.6}$

$\boldsymbol{A}$ be the event that it will rain tomorrow. $P\left(\frac{A}{E_1}\right) = 0.8$ & $P\left(\frac{A}{E_2}\right) = 0.7$ $\mathbf{\frac{1}{2}}$

We have, $P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74.$

The probability of rain tomorrow is $\mathbf{0.74}$. $\mathbf{\frac{1}{2}}$

OR

31.(b)

Given $P(X = r)\alpha\frac{1}{5^r}$ $\mathbf{\frac{1}{2}}$

$P(X = r) = k\frac{1}{5^r},$ (where $\boldsymbol{k}$ is a non-zero constant)

$P(r = 0) = k.\frac{1}{5^0}$

$P(r = 1) = k.\frac{1}{5^1}$

$P(r = 2) = k.\frac{1}{5^2}$ $\mathbf{\frac{1}{2}}$

$P(r = 3) = k.\frac{1}{5^3}$

…………………………

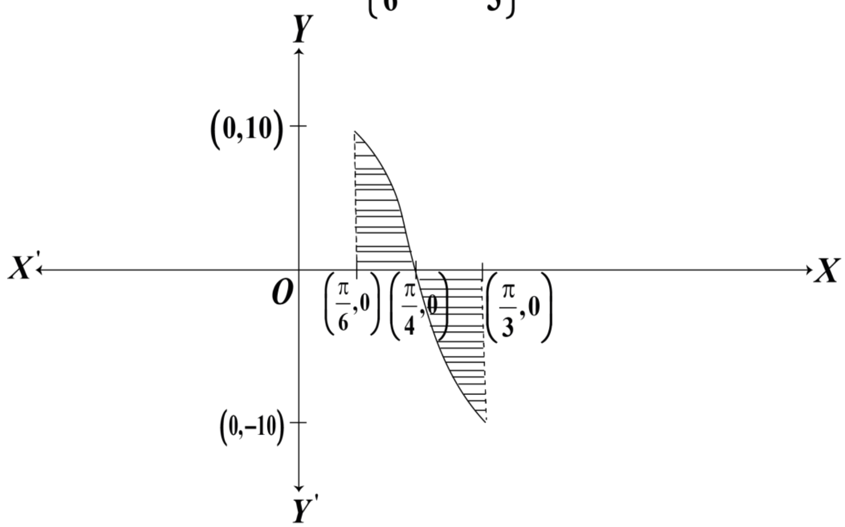
…………………………

We have, $P(X = 0) + P(X = 1) + P(X = 2) + \ldots\ldots\ldots\ldots = 1$ $\mathbf{\frac{1}{2}}$

	$\Rightarrow k\left(1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\ldots\ldots\ldots\right)=1$ $\Rightarrow k\left(\frac{1}{1-\frac{1}{5}}\right)=1 \Rightarrow k=\frac{4}{5}$ So, $P(X<3)=P(X=0)+P(X=1)+P(X=2)$ $=\frac{4}{5}\left(1+\frac{1}{5}+\frac{1}{5^2}\right)=\frac{4}{5}\left(\frac{25+5+1}{25}\right)=\frac{124}{125}.$	**$\frac{1}{2}$** **1**

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.	$y=20\cos 2x\ ;\ \left\{\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$  Required area $=20\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}cos2x\,dx+\left\|20\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}cos2x\,dx\right\|$ $=20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}+\left\|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right\|$ $=10\left(1-\frac{\sqrt{3}}{2}\right)+\ 10\left(1-\frac{\sqrt{3}}{2}\right)=20\left(1-\frac{\sqrt{3}}{2}\right)$ sq. units.	**1** **1+1** **1** **1**
33.	$\boldsymbol{y=ax^2+bx+c}$ $\boldsymbol{15=4a+2b+c}$ $\boldsymbol{25=16a+4b+c}$ $\boldsymbol{15=196a+14b+c}$ The set of equations can be represented in the matrix form as $\boldsymbol{AX=B}$, where $A=\begin{bmatrix}4&2&1\\16&4&1\\196&14&1\end{bmatrix}$, $X=\begin{bmatrix}a\\b\\c\end{bmatrix}$ and $B=\begin{bmatrix}15\\25\\15\end{bmatrix}\Rightarrow\begin{bmatrix}4&2&1\\16&4&1\\196&14&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}15\\25\\15\end{bmatrix}$. $\|A\|=4(4-14)-2(16-196)+(224-784)=-40+360-560=-240\neq 0.$ Hence $\boldsymbol{A^{-1}}$ exists.	**1** **$\frac{1}{2}$** **$\frac{1}{2}$**

	Now, $adj(A) = \begin{bmatrix} -10 & 180 & -560 \\ 12 & -192 & 336 \\ -2 & 12 & -16 \end{bmatrix}^T = \begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}$	1
	$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} 24 \\ -384 \\ -48 \end{bmatrix}$	1
	$\therefore \boldsymbol{a = -\frac{1}{2}, b = 8, c = 1}$	$\frac{1}{2}$
	So, the equation becomes $\boldsymbol{y = -\frac{1}{2}x^2 + 8x + 1}$	$\frac{1}{2}$
34.(a)	We have, $f(x) = \lvert x \rvert^3, \begin{cases} x^3, \text{if } x \geq 0 \\ (-x)^3 = -x^3, \text{if} x < 0 \end{cases}$	
	Now, $(LHD\ at\ x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \left(\frac{-x^3 - 0}{x}\right) = \lim_{x \to 0^-} (-x^2) = 0$	$\frac{1}{2}$
	$(RHD\ atx = 0) \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \left(\frac{x^3 - 0}{x}\right) = \lim_{x \to 0} (-x^2) = 0$	$\frac{1}{2}$
	$\therefore (LHD\ of\ f(x)\ at\ x = 0) = (RHD\ of\ f(x)\ at\ x = 0)$	$\frac{1}{2}$
	So, $\boldsymbol{f(x)}$ is differentiable at $\boldsymbol{x = 0}$ and the derivative of $\boldsymbol{f(x)}$ is given by	
	$f'(x) = \begin{cases} 3x^2, \text{if} x \geq 0 \\ -3x^2, \text{if} x < 0 \end{cases}$	1
	Now, $(LHDoff'(x)atx = 0) = \lim_{x \to 0^-} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^-} \left(\frac{-3x^2 - 0}{x}\right) = \lim_{x \to 0^-} (-3x) = 0$	$\frac{1}{2}$
	$(RHD\ of f'(x)\ at\ x = 0) = \lim_{x \to 0^+} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^+} \left(\frac{3x^2 - 0}{x - 0}\right) = \lim_{x \to 0^+} (3x) = 0$	$\frac{1}{2}$
	$\therefore (LHD\ of f'(x) at\ x = 0) = (RHDoff'(x)atx = 0)$	$\frac{1}{2}$
	So, $f'(x)$ is differentiable at $\boldsymbol{x = 0.}$	$\frac{1}{2}$
	Hence, $f''(x) = \begin{cases} 6x, \text{if} x \geq 0 \\ -6x, \text{if} x < 0. \end{cases}$	$\frac{1}{2}$
OR **34 .(b)**	Given relation is $(x - a)^2 + (y - b)^2 = c^2, c > 0.$	
	Let $\boldsymbol{x - a = c\cos\theta}$ and $y - b = c \sin\theta.$	$\frac{1}{2}$
	Therefore, $\frac{dx}{d\theta} = -c \sin\theta$ And $\frac{dy}{d\theta} = c \cos\theta$	$\frac{1}{2}$
	$\therefore \frac{dy}{dx} = -\cot\theta$	1
	Differentiate both sides with respect to θ, we get $\frac{d}{d\theta}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}(-\cot\theta)$	$\frac{1}{2}$

Or, $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{d\theta} = cos\,e\,c^2\theta$

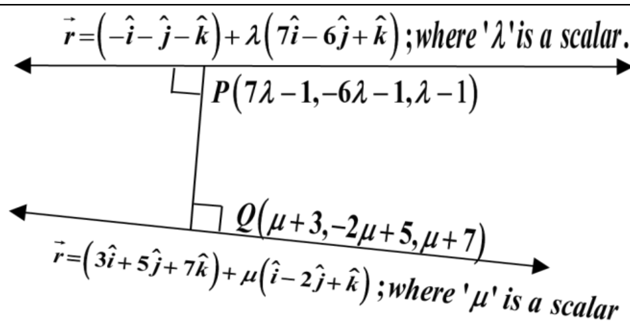
Or, $\frac{d^2y}{dx^2}(-c\,sin\,\theta) = \text{cosec}^2\theta$ **$\frac{1}{2}$**

$$\frac{d^2y}{dx^2} = -\frac{cosec^3\theta}{c}$$ **$\frac{1}{2}$**

$$\therefore \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{c[1+cot^2\theta]^{\frac{3}{2}}}{-cos\,ec^3\theta} = \frac{-c(cos\,ec^2\theta)^{\frac{3}{2}}}{\text{cosec}^3\theta} = -c,$$ **1**

Which is constant and is independent of $\boldsymbol{a}$ and $\boldsymbol{b}$. **$\frac{1}{2}$**

35.(a)



Given that equation of lines are

$\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$................$(i)$ and

$\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$..................$(ii)$

The given lines are non-parallel lines as vectors $7\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ are not parallel. There is a unique line segment $\boldsymbol{PQ}$ ($\boldsymbol{P}$ lying on line $(\boldsymbol{i})$ and $\boldsymbol{Q}$ on the other line $(\boldsymbol{ii})$), which is at right angles to both the lines $\boldsymbol{PQ}$ is the shortest distance between the lines.

Hence, the shortest possible distance between the lines $= \boldsymbol{PQ}$.

Let the position vector of the point $\boldsymbol{P}$ lying on the line $\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$ where '$\boldsymbol{\lambda}$' is a scalar, is $(7\lambda - 1)\hat{\imath} - (6\lambda + 1)\hat{\jmath} + (\lambda - 1)\hat{k}$, for some $\boldsymbol{\lambda}$ and the position vector of the point $\boldsymbol{Q}$ lying on the line $\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ where '$\boldsymbol{\mu}$' is a scalar, is $(\mu + 3)\hat{\imath} + (-2\mu + 5)\hat{\jmath} + (\mu + 7)\hat{k}$, for some $\boldsymbol{\mu}$. Now, the vector **$\frac{1}{2}$** **$\frac{1}{2}$**

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (\mu + 3 - 7\lambda + 1)\hat{\imath} + (-2\mu + 5 + 6\lambda + 1)\hat{\jmath} + (\mu + 7 - \lambda + 1)\hat{k}$$

$i.e., \overrightarrow{PQ} = (\mu - 7\lambda + 4)\hat{\imath} + (-2\mu + 6\lambda + 6)\hat{\jmath} + (\mu - \lambda + 8)\hat{k}$; (where '$\boldsymbol{O}$' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{\imath} - 6\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + \hat{k}$. **1**

$\Rightarrow (\mu - 7\lambda + 4).7 + (-2\mu + 6\lambda + 6).(-6) + (\mu - \lambda + 8).1 = 0$

$\&(\mu - 7\lambda + 4).1 + (-2\mu + 6\lambda + 6).(-2) + (\mu - \lambda + 8).1 = 0$

$\Rightarrow \mathbf{20\mu - 86\lambda = 0} \Rightarrow \mathbf{10\mu - 43\lambda = 0} \,\&\, 6\mu - 20\lambda = 0 \Rightarrow 3\mu - 10\lambda = 0$ **1**

On solving the above equations, we get $\boldsymbol{\mu = \lambda = 0}$ **$\frac{1}{2}$**

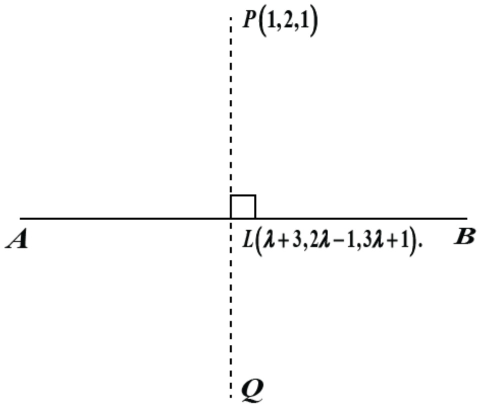
So, the position vector of the points ***P*** and ***Q*** are $-\hat{i} - \hat{j} - \hat{k}$ and $3\hat{i} + 5\hat{j} + 7\hat{k}$ respectively.

$\overrightarrow{PQ} = 4\hat{i} + 6\hat{j} + 8\hat{k}$ and **$\frac{1}{2}$**

$|\overrightarrow{PQ}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116} = 2\sqrt{29}\, units.$ **1**

OR

35.(b)



Let $\boldsymbol{P(1, 2, 1)}$ be the given point and ***L*** be the foot of the perpendicular from ***P*** to the given line ***AB*** $(as\ shown\ in\ the\ figure\ above)$.

Let's put $\boldsymbol{\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3} = \lambda}$. Then, $\boldsymbol{x = \lambda + 3, y = 2\lambda - 1, z = 3\lambda + 1}$ **$\frac{1}{2}$**

Let the coordinates of the point ***L*** be $\boldsymbol{(\lambda + 3, 2\lambda - 1, 3\lambda + 1)}$.

So, direction ratios of ***PL*** are $(\lambda + 3 - 1, 2\lambda - 1 - 2, 3\lambda + 1 - 1)$ $i.e., (\lambda + 2, 2\lambda - 3, 3\lambda)$

Direction ratios of the given line are **1, 2 and 3,** which is perpendicular to ***PL***. Therefore, we have, **$\frac{1}{2}$**

$$(\lambda + 2).1 + (2\lambda - 3).2 + 3\lambda.3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = \frac{2}{7}$$ **$\frac{1}{2}$**

Then, $\lambda + 3 = \frac{2}{7} + 3 = \frac{23}{7}$; $2\lambda - 1 = 2\left(\frac{2}{7}\right) - 1 = -\frac{3}{7}$; $3\lambda + 1 = 3\left(\frac{2}{7}\right) + 1 = \frac{13}{7}$ **$\frac{1}{2}$**

Therefore, coordinates of the point ***L*** are $\left(\frac{23}{7}, -\frac{3}{7}, \frac{13}{7}\right)$.

Let $Q(x_1, y_1, z_1)$ be the image of $\boldsymbol{P(1, 2, 1)}$ with respect to the given line. Then, ***L*** is the mid-point of ***PQ***. **1**

Therefore, $\frac{1+x_1}{2} = \frac{23}{7}, \frac{2+y_1}{2} = -\frac{3}{7}, \frac{1+z_1}{2} = \frac{13}{7} \Rightarrow x_1 = \frac{39}{7}, y_1 = -\frac{20}{7}, z_1 = \frac{19}{7}$

Hence, the image of the point $\boldsymbol{P(1,2,1)}$ with respect to the given line $Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right)$. **1**

The equation of the line joining $\boldsymbol{P(1,2,1)}$ and $Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right)$ is

	$\frac{x-1}{32/7}=\frac{y-2}{-34/7}=\frac{z-1}{12/7} \Rightarrow \frac{x-1}{16}=\frac{y-2}{-17}=\frac{z-1}{6}$.	1

Section –E

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36.	(i) $V=(40-2x)(25-2x)xcm^3$	1
	(ii) $\frac{dV}{dx}=4(3x-50)(x-5)$	1
	(iii) (a) For extreme values $\frac{dV}{dx}=4(3x-50)(x-5)=0$	$1/2$
	$\Rightarrow x=\frac{50}{3}$ or $x=5$	$1/2$
	$\frac{d^2V}{dx^2}=24x-260$	$1/2$
	$\therefore \frac{d^2V}{dx^2}$ at $x=5$ is $-140<0$	$1/2$
	$\therefore V$ is max $when\ x=5$	
	(iii) **OR**	
	(b) For extreme values $\frac{dV}{dx}=4(3x^2-65x+250)$	$1/2$
	$\frac{d^2V}{dx^2}=4(6x-65)$	$1/2$
	$\frac{dV}{dx}\ at\ x=\frac{65}{6}$ exists and $\frac{d^2V}{dx^2}\ at\ x=\frac{65}{6}\ is\ 0$.	
	$\frac{d^2V}{dx^2}\ at\ x=\left(\frac{65}{6}\right)^-$ is negative and $\frac{d^2V}{dx^2}\ at\ x=\left(\frac{65}{6}\right)^+$ is positive	$1/2$
	$\therefore x=\frac{65}{6}$ is a point of inflection.	$1/2$
37.	**(i)** **Number of relations is equal to the number of subsets of the set $B\times G=2^{n(B\times G)}$ $=2^{n(B)\times n(G)}=2^{3\times 2}=2^6$** ($\boldsymbol{Where n(A)\ denotes\ the\ number\ of\ the\ elements\ in\ the\ finite\ set\ A}$)	1
	(ii) **Smallest Equivalence relation on G is** $\{(g_1,g_1),(g_2,g_2)\}$	1
	(iii) (a) (A) reflexive but not symmetric = $\{(b_1,b_2),(b_2,b_1),(b_1,b_1),(b_2,b_2),(b_3,b_3),(b_2,b_3)\}$.	

So the minimum number of elements to be added are

$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$ **1**

{Note : it can be any one of the pair from**,** (b_3, b_2), (b_1, b_3), (b_3, b_1) **in place of** (b_2, b_3) ***also*}**

(B) reflexive and symmetric but not transitive =

{$(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$ }.

1

So the minimum number of elements to be added are

$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$**,** (b_3, b_2)

OR (iii) (b) One-one and onto function

$x^2 = 4y$. let $y = f(x) = \frac{x^2}{4}$

Let $x_1, x_2 \in [0, 20\sqrt{2}]$ such that $f(x_1) = f(x_2) \Rightarrow \frac{{x_1}^2}{4} = \frac{{x_1}^2}{4}$ **1**

$\Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in [0, 20\sqrt{2}]$

$\therefore$ f is one-one function

Now, $0 \le y \le 200$ hence the value of y is non-negative

and $f(2\sqrt{y}) = y$

$\therefore$ for any arbitrary $y \in [0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$ **1**

hence f is onto function.

38. Let E_1 be the event that one parrot and one owl flew from cage –I

E_2 be the event that two parrots flew from Cage-I

A be the event that the owl is still in cage-I

(i) Total ways for A to happen

From cage I 1 parrot and 1 owl flew and then from Cage-II 1 parrot and 1 owl flew back + From cage I 1 parrot and 1 owl flew and then from Cage-II 2 parrots flew back + From cage I 2 parrots flew and then from Cage-II 2 parrots came back. **$\frac{1}{2}$**

$=(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})$

Probability that the owl is still in cage –I = P($E_1 \cap A$) + P($E_2 \cap A$)

$$\frac{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_2})(8_{C_2})}{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})}$$ **1**

$$= \frac{35 + 280}{35 + 105 + 280} = \frac{315}{420} = \frac{3}{4}$$ **$\frac{1}{2}$**

	(i) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in cage-I is $P\left({E_1}/{A}\right)$	$\frac{1}{2}$
	$P\left({E_1}/{A}\right) = \frac{P(E_1 \cap A)}{P(E_1 \cap A)+P(E_2 \cap A)}$ (by Baye's Theorem)	$\frac{1}{2}$
	$= \frac{\frac{35}{420}}{\frac{315}{420}} = \frac{1}{9}$	1